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The jamming transition of granular media

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Abstract. A statistical mechanical approach to granular material is proposed. Using lattice models from standard statistical mechanics and results from a mean-field replica approach we find a jamming transition in granular media closely related to the glass transition in supercooled liquids. These models reproduce the logarithmic relaxation in granular compaction and reversible–irreversible lines, in agreement with experimental data. The models also exhibit aging effects and breakdown of the usual fluctuation-dissipation relation. It is shown that the glass transition may be responsible for the logarithmic relaxation and may be related to the cooperative effects underlying many phenomena exhibited by granular materials such as the Reynolds transition.

(Some figures in this article appear in colour in the electronic version; see www.iop.org)

1. Introduction

Despite their importance for industrial applications, non-thermal disordered systems such as granular media have only recently begun to be systematically studied by the physics community [1]. In particular, concepts from statistical mechanics seem to be successful for describing these systems, as suggested in his pioneering work by Edwards [2]. In fact, granular media are composed of a large number of single grains and, just as for the systems of standard statistical mechanics, each of their macroscopic states corresponds to a huge number of microstates. Furthermore, they show very general reproducible macroscopic behaviours whose general properties are not material specific and which are statistically characterized by very few control parameters, such as their density and load and the amplitude of the external drive [1]. Granular media are 'non-thermal' systems since thermal energy plays no role with respect, for instance, to gravitational energies involved in grain displacements; however, thermal motion can be replaced by agitation induced by shaking or other driving mechanisms and this allows the system to explore its space of configurations.

A 'tapping' dynamics has several control parameters. One that appears to be very important, and is related to the amplitude of the vibrations of the external driving, is the adimensional ratio $\Gamma_{ex} = a/g$ [1, 3], where a is the shaking peak acceleration and g the constant acceleration due to gravity. It has been suggested that Γ_{ex} plays a role very similar to that of 'temperature' in thermal systems. For simplicity, below we suppose the characteristic frequency, ω, of the external driving force to be fixed, and in fact in the typical experiments such as those in references [3, 4] in which we are interested, it plays a minor role with respect to Γ_{ex} (see [1]). Material parameters such as restitution, friction coefficients, and the presence

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of moisture also usually have to be considered, but they generally do not affect the overall behaviours.

Experimentally it is found that at low Γ_{ex}, relaxation processes such as density compaction under tapping are logarithmically slow. The essential ingredient in the dynamics of dense granular media is the presence of mechanisms 'frustrating' the motion of grains due to steric hindrance and hard-core repulsion. We have introduced 'frustrated lattice gases' to describe the dynamics of granular materials in the regime of high densities or not too strong shaking [5–7]. They interestingly allow one to interpret, in a single framework, several different properties of granular media ranging from logarithmic compaction, density fluctuations, 'irreversible–reversible' cycles, aging, breakdown of the fluctuation-dissipation relation, and segregation to avalanche effects, the Reynolds transition, and several others [6, 7, 13, 14, 25]. In particular they predict the existence of a definite 'jamming' transition in granular media, in strict correspondence with the glass transition of glass formers and spin glasses [1, 8–10]. Furthermore, they constitute new important applications of statistical mechanics to powders [2, 11].

2. Frustrated lattice gas models for granular media

The very simple schematic models for gently shaken granular media that we introduced are based on a drastic reduction of the degrees of freedom of the systems to those we suppose to be essential. The models each consist of a system of elongated grains which move on a lattice. Grains are subject to gravity and, eventually, to shaking, which is simulated with a driven diffusion like Monte Carlo dynamics. The crucial ingredient of the models is the presence of geometric constraints on the motion of the grains; thus, they have been called frustrated lattice gases [5, 6]. Particles in our models occupy the sites of, say, a square (or cubic) lattice (see figure 1). They also each have an internal degree of freedom $S_i = \pm 1$ corresponding to the possible 'orientations' along the two lattice axes. Two nearest-neighbour sites can be both occupied only if the particles do not overlap.

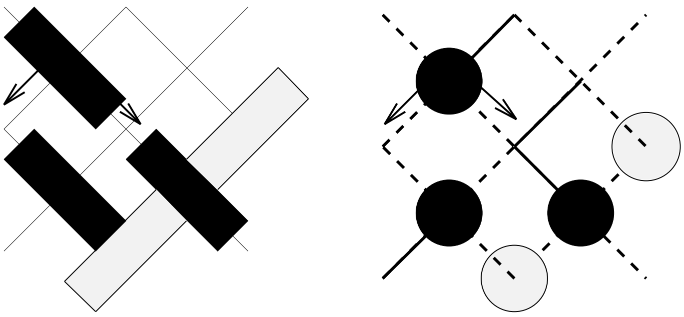

Figure 1. A schematic picture of the two kinds of frustrated lattice gas model described in the text. Left: the Tetris model. Right: the Ising frustrated lattice gas, IFLG. Straight and dashed lines represent the two kinds of interaction $\epsilon_{ij} = \pm 1$. Filled circles represent particles with 'orientation' $S_i = \pm 1$ (black/white).

Each 'experimental' tap can be divided into two parts: one where the average kinetic energy of the grains is finite; and a second where it goes to zero. Thus, in our models grains undergo a schematic driven diffusive Monte Carlo (MC) dynamics: in the absence of vibrations they are subject only to gravity and they can only move downwards, always fulfilling the non-overlap condition; the presence of vibration is introduced by allowing the particles to diffuse with a probability p_{up} of moving upwards and a probability $p_{down} = 1 - p_{up}$ of moving downwards. The quantity $-\ln(x_0)/2$, with $x_0 = p_{up}/p_{down}$, as we will see, plays the role of an effective temperature and can be related to the experimental tap vibration intensity, Γ_{ex}.

2.1. Hamiltonian description

The general model introduced above can be described in terms of a standard lattice gas of statistical mechanics [5, 6]. The system Hamiltonian must have a hard-core repulsion term ($J \to \infty$):

$$H_{HC} = J \sum_{\langle ij \rangle} f_{ij}(S_i, S_j) n_i n_j. \tag{1}$$

Here $n_i = 0, 1$ are occupancy variables describing the positions of grains, $S_i = \pm 1$ are 'spin' variables associated with the orientations of the particles, J represents the infinite repulsion felt by the particles when they have the wrong orientations. The hard-core repulsion function $f_{ij}(S_i, S_j)$ is 0 or 1 depending on whether the configuration S_i, S_j is right (allowed) or wrong (not allowed); see figure 1.

The choice of $f_{ij}(S_i, S_j)$ depends on the particular model. In particular, here we consider two models: the Tetris model and the Ising frustrated lattice gas (IFLG). The Tetris model is made up of elongated particles (see figure 1), each of which may point in two (orthogonal) directions coinciding with the two lattice bond orientations. In this case $f_{ij}(S_i, S_j)$ is given by [7]

$$f_{ij}^{\text{Tetris}}(S_i, S_j) = (1/2)(S_i S_j - \epsilon_{ij}(S_i + S_j) + 1)$$

where $\epsilon_{ij} = +1$ for bonds along one direction of the lattice and $\epsilon_{ij} = -1$ for bonds along the other. In order for it to have a non-trivial behaviour, the dynamics of the Tetris model has imposed on it a crucial *purely kinetic constraint*: particles can flip their 'spin' only if three of their own neighbouring sites are empty.

A real granular system may contain more disorder due to the presence of a wider grain shape distribution or to the absence of a regular underlying lattice. Typically, each grain moves in the disordered environment generated by its neighbours. In order to schematically consider these effects within the above context, another kind of 'frustrated lattice gas' was introduced, made up of grains moving in a lattice with quenched geometric disorder. Such a model, the Ising frustrated lattice gas (IFLG), has the following hard-core repulsion function, $f_{ij}(S_i, S_j)$ [6]:

$$f_{ij}^{\text{IFLG}}(S_i, S_j) = (1/2)(\epsilon_{ij} S_i S_j - 1)$$

where $\epsilon_{ij} = \pm 1$ are quenched random interactions associated with the edges of the lattice, representing the fact that particles must satisfy the geometric constraint of the environment considered as 'practically' quenched (see figure 1). The IFLG shows a non-trivial dynamics *without* the necessity of introducing kinetic constraints.

The phase diagram of the Tetris Hamiltonian corresponds to the usual antiferromagnetic Ising model with dilution. The Hamiltonian of the IFLG exhibits richer behaviours. In the limit where all sites are occupied ($n_i = 1\ \forall i$), it becomes equal to the usual $\pm J$ Ising Edwards–Anderson spin glass [10]. In the limit $J \to \infty$ (which we consider below), a version of site-frustrated percolation is recovered [15, 16].

The other important contribution to the full Hamiltonian of a granular pack that we consider below must be gravitational energy: $\mathcal{H} = H_{HC} + H_G$, where

$$H_G = g \sum_i n_i y_i$$

and g is the gravity constant and y_i is the height of particle i (the mass of the grain and the lattice spacing are set to unity). The temperature, T, of the present Hamiltonian system (with $J = \infty$) is related to the ratio $x_0 = p_{up}/p_{down}$ via the following relation: $e^{-2g/T} = x_0$. It is useful to define the adimensional quantity $\Gamma \equiv 1/\ln(x_0^{-1/2}) = T/g$, which we assume plays the same role as the amplitude of the vibrations in real granular matter—that is, Γ is a smooth function of Γ_{ex}.

It is easy to show [25] that if the system reaches equilibrium, the temperature T is

$$T^{-1} = \frac{\partial \ln \Omega}{\partial E} \tag{2}$$

where Ω is the number of configurations corresponding to the gravitational energy E. Note that for samples with constant particle density, E is proportional to the volume and thus $1/T$ coincides with Edwards' compactivity [2] apart from a proportionality constant.

3. The dynamics of compaction

To describe experimental observations regarding the grain density relaxation under a sequence of taps, a logarithmic law was proposed in reference [3]:

$$\rho(t_n) = \rho_\infty - \Delta\rho_\infty/[1 + B \ln(t_n/\tau_1 + 1)]. \tag{3}$$

This law has proved to be satisfied very well by relaxation data in the IFLG model [6], which can be excellently rescaled with experimental data. In figure 2, MC compaction data for four different amplitudes, Γ, as well as the experimental data for three different amplitudes, Γ_{ex}, are collapsed onto a single curve using equation (3). The agreement is very satisfactory (details regarding these data are given in reference [6]).

Interestingly, the results from the Tetris model are similar [7], but the asymptotic density ρ_∞ is numerically indistinguishable from 1, and thus almost independent of x_0, a fact in contradiction with both IFLG and experimental results [3].

4. Glassy behaviours of granular media

We have seen that compaction shows extremely long relaxation times. In fact, we show that granular systems are typically in off-equilibrium configurations and that they may undergo a 'jamming' transition. In particular, in the same framework of the above models, we show how experimental results on 'memory' effects (the so-called 'irreversible–reversible' cycles) indicate the existence of a dynamical glassy transition, which can be defined in a similar way to the glassy transition in real glass formers.

It has been shown in the mean-field approximation [17] and numerically for finite-dimensional systems [16], that the IFLG, in the absence of gravity, exhibits a spin-glass (SG) transition at high density (or low temperature) similar to the one found in the p-spin model. Arenzon has extended the mean-field solution of the IFLG model to include the presence of gravity [18], showing that at low Γ the system is frozen in a SG-like phase, but at higher Γ it separates into a frozen SG phase at the bottom and a fluid phase on top. Above a critical Γ, the system is entirely fluid [19].

4.1. 'Irreversible–reversible' cycles and the dynamical glass transition

Tapping experiments typically show 'irreversible–reversible' cycles [4] (see figure 3): during a sequence of taps, if the system is successively shaken at increasing vibration amplitudes, its bulk

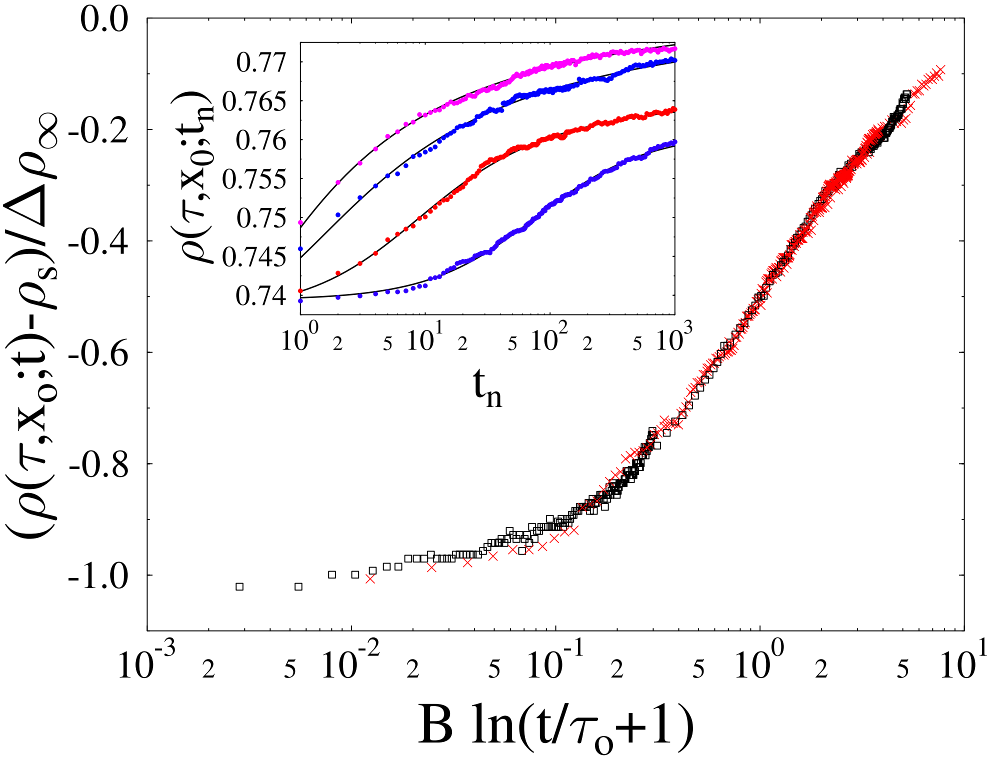

Figure 2. The compaction of granular media at low shaking amplitudes. Experimental data from Knight *et al* (squares) and MC data for the IFLG (circles) rescaled according equation (3). Inset: density $\rho(\tau, x_0; t_n)$ from MC data as a function of tap number t_n, for tap amplitudes $x_0 = 0.001, 0.01, 0.05, 0.1$ (from bottom to top) and duration $\tau = 3.28 \times 10^1$. The superimposed curves are logarithmic fits to equation (3).

density typically grows, as in compaction, and then, after a characteristic Γ_{ex}, starts decreasing. However, if the amplitude of shaking is decreased back to zero, the density generally does not follow the same path since it keeps growing. In this sense these observations indicate the existence of 'memory effects'.

In analogy with real experiments, cycles of taps were performed in the IFLG model in which the vibration amplitude Γ was varied in a sequence of increments $\gamma = \Delta\Gamma/\tau$, with constant tap duration τ. After each tap, the static bulk density of the system $\rho(\Gamma_n)$ (n is the number of the nth tap) was measured. The data are qualitatively very similar to those reported from real experiments on dry granular packs [4]. Furthermore, they allow one to define a 'jamming' transition point $\Gamma_g(\gamma)$ in analogy with the glass transition in glass formers [13], as explained in figure 4.

4.2. Two-time correlation functions

Granular media in the above dynamical situations show 'aging' too [13]. To see this, just as for glassy systems, it is useful to consider the two-time correlation functions as ($t \geqslant t'$)

$$C(t,t') = \frac{[\langle\rho(t)\rho(t')\rangle - \langle\rho(t)\rangle\langle\rho(t')\rangle]}{[\langle\rho(t')^2\rangle - \langle\rho(t')\rangle^2]}$$

where $\rho(t)$ is the bulk density of the system at time t.

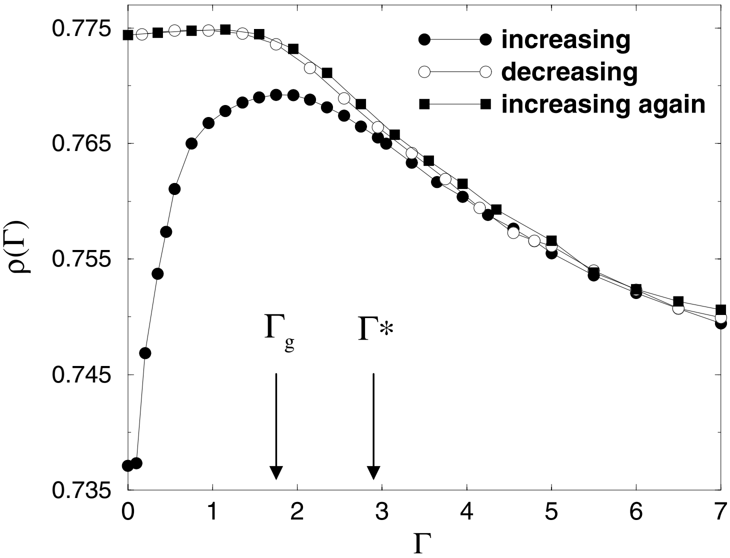

Figure 3. The static bulk density, $\rho(\Gamma)$, of the IFLG model as a function of the vibration amplitude, Γ, in cyclic vibration sequences. The system is shaken with an amplitude Γ which at first is increased (filled circles), then is decreased (empty circles), and, finally, is increased again (filled squares) with a given 'annealing–cooling' velocity $\gamma \equiv \Delta\Gamma/\tau$. Here we fixed $\gamma = 1.25 \times 10^{-3}$. The upper part of the cycle is approximately 'reversible' (i.e., empty circles and filled squares fall roughly on the same curve). The data compare rather well with the experimental data of Novak *et al*. Γ^* is approximately the point where the 'irreversible' and the 'reversible' branches meet. Γ_g signals the location of a 'jamming transition'.

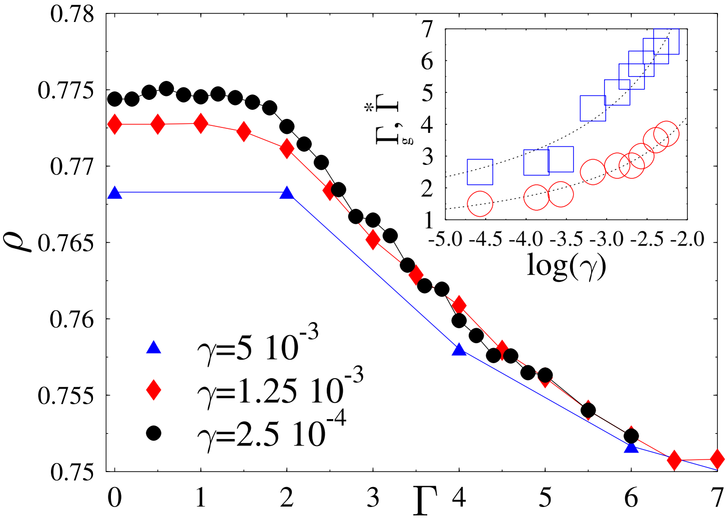

Figure 4. Main figure: as in figure 3, the density, $\rho(\Gamma)$, is plotted as a function of the vibration amplitude, Γ for three values of the 'cooling' velocity γ. Here only the descending parts of the cycles are shown. As for glasses, a too-fast cooling drives the system out of equilibrium. The position of the shoulder, $\Gamma_g(\gamma)$, schematically indicates a 'jamming transition'. Inset: the numerical estimate, in the IFLG model, of the dependence of $\Gamma_g(\gamma)$ (circles) and $\Gamma^*(\gamma)$ (squares) on the cooling rate γ. Superimposed are logarithmic fits in analogy with those for the glass transition temperature, $T_g(\gamma)$, in glasses [4]. When $\gamma \to 0$, we roughly find $\Gamma_g(0) = \Gamma^*(0)$.

The fit, at low Γ, of the two-time correlation function, $C(t, t')$ (over five decades of MC time), reveals the following interesting approximate scaling form:

$$C(t, t') = (1 - c_\infty)\frac{\ln[(t' + t_s)/\tau]}{\ln[(t + t_s)/\tau]} + c_\infty \qquad (4)$$

where τ (which is no longer the 'tap duration', as it was before); t_s and c_∞ are fit parameters. It is very interesting that the above behaviour is found in both the models discussed (Tetris and IFLG) [13]. The data for the two models, for several values of Γ, rescaled onto a single universal master function, are plotted in figure 5. It is interesting that such scaling behaviours occur in the off-equilibrium dynamics of apparently different systems [24].

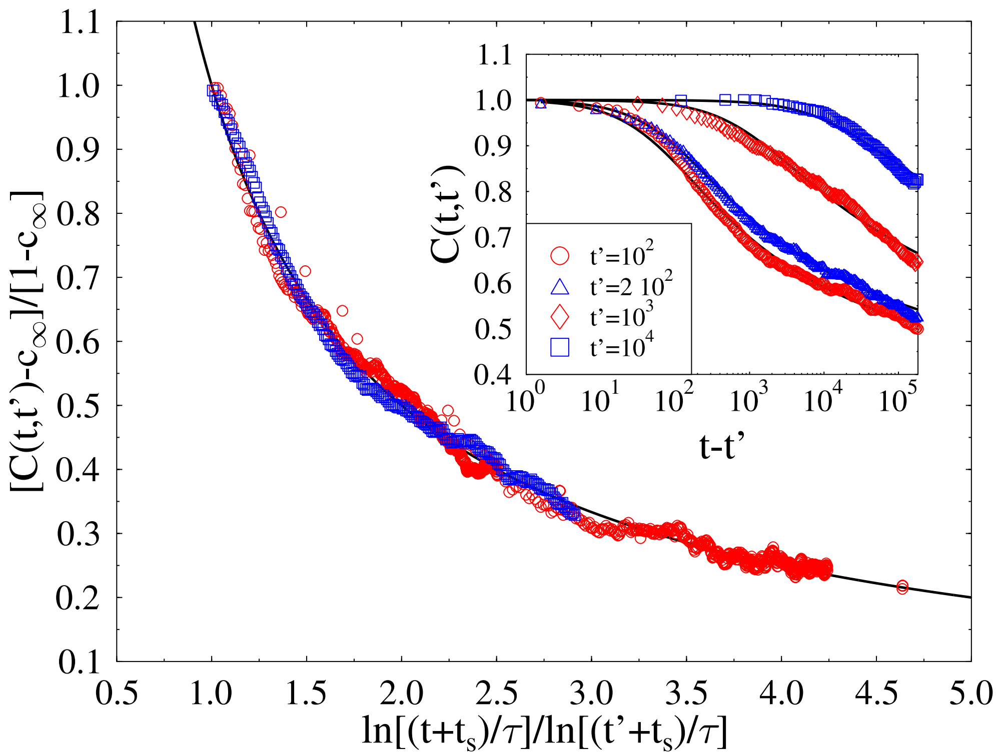

Figure 5. The two-time density–density correlation function, $(C(t, t') - c_\infty)/(1 - c_\infty)$, as a function of the scaling variable $\alpha = \ln[(t + t_s)/\tau]/\ln[(t' + t_s)/\tau]$. Scaled onto the same master function are data from both models considered in the present paper (Tetris (squares) and IFLG (circles)) for $\Gamma = -1/\ln(x_0)$ with $x_0 \in [10^{-4}, 10^{-1}]$. Inset: the correlation $C(t, t')$ for the Tetris model at $\Gamma = 0.22$ (or $x_0 = 0.01$) as a function of $t - t'$ for $t' = 10^2, 2 \times 10^2, 10^3, 10^4$.

4.3. *Response functions and the 'fluctuation-dissipation' relation*

In reference [14] it was argued that in granular media it is possible to formulate a link between the response and fluctuations, i.e., the analogue of a 'fluctuation-dissipation theorem' (FDT) (see [20]), where the amplitude of external vibrations plays the role of the usual 'temperature'. In typical situations, the FDT coincides neither with its usual version at equilibrium nor with the extensions valid in off-equilibrium thermal systems in the so-called 'small-entropy-production' limit [21]. The origin of the universalities in the off-equilibrium dynamics in the above heterogeneous classes of materials is still unknown.

Our systems are 'shaken' at a given amplitude x_0 and the average grain height:

$$h_0(t) = \langle H(t) \rangle \qquad \left(\text{with } H(t) = \sum_i y_i(t)/N \right)$$

recorded as long as the 'mean square displacement'

$$B(t, t') \equiv \langle [H(t) - H(t')]^2 \rangle.$$

To measure the response function, the average height, $h_1(t, t_w)$, was also recorded in an identical copy of the system (a 'replica') perturbed by a small increase of the shaking amplitude, after a time t_w.

The difference in height between the perturbed and unperturbed systems $\Delta h(t, t_w) = h_1(t, t_w) - h_0(t)$ is by definition the integrated response. FDT and its generalizations concern the relation between $\Delta h(t, t_w)$ and the displacement, $B(t, t_w)$. In analogy with thermal systems [9], in reference [14] it was proposed that

$$\Delta h(t, t_w) \simeq \frac{X}{2} \Delta(\Gamma^{-1}) B(t, t_w). \tag{5}$$

Equation (5) states that, after transients, in a granular system the measure of the height variation after a change in shaking amplitude should be proportional to the 'displacement' recorded during an unperturbed run. In the simplest situations, such as for equilibrium thermal systems, the proportionality factor, X, is a constant, but, more generally, X is function of t_w and t themselves. Interestingly, in glassy systems the quantity $g\Gamma/X$ has the meaning of an 'effective temperature' [22, 23].

In the present model, equation (5) seems to be approximately valid, as it also is if in typical off-equilibrium situations X slowly depends on Γ and t_w. This is shown, for high x_0, in the top panels of figure 6: in the long-time regime, X is equal to 1, showing that, in the high-'temperature' and low-density region, the usual equilibrium version of FDT is obeyed. In the low-x_0 region, the above picture changes, as shown in the bottom panels of figure 6: after an early transient, the response, Δh, is *negative* and, thus, X is negative, which would asymptotically correspond to a negative 'effective temperature'.

5. Conclusions

In conclusion, the present paper has dealt with the understanding of the 'jamming' transition in granular media via the introduction of models from standard statistical mechanics for describing these 'non-thermal' systems (the IFLG and the Tetris models [6, 7]).

They exhibit a logarithmic compaction when subjected to gentle shaking in the presence of gravity, a compaction extremely close to what is experimentally observed in granular packs [3]. The presence of such slow dynamics is linked to the existence of a 'jamming' transition in granular media. In fact, in the high-density region, at ρ_m, the models undergo a structural arrest where the grain self-diffusivity becomes zero [6].

Self-diffusion suppression at ρ_m signals that, above such a density, it is impossible to obtain a macroscopic rearrangement of grains without increasing the system volume, a feature interestingly similar to the phenomenon of the Reynolds dilatancy transition [6].

A very important fact is that the results from the present models are in excellent agreement with known experimental ones, and the many new predictions of the models must be experimentally investigated. We have also discussed the intriguing connections of granular media with other materials, such as glassy systems and spin glasses, where geometrical disorder and frustration play a crucial role.

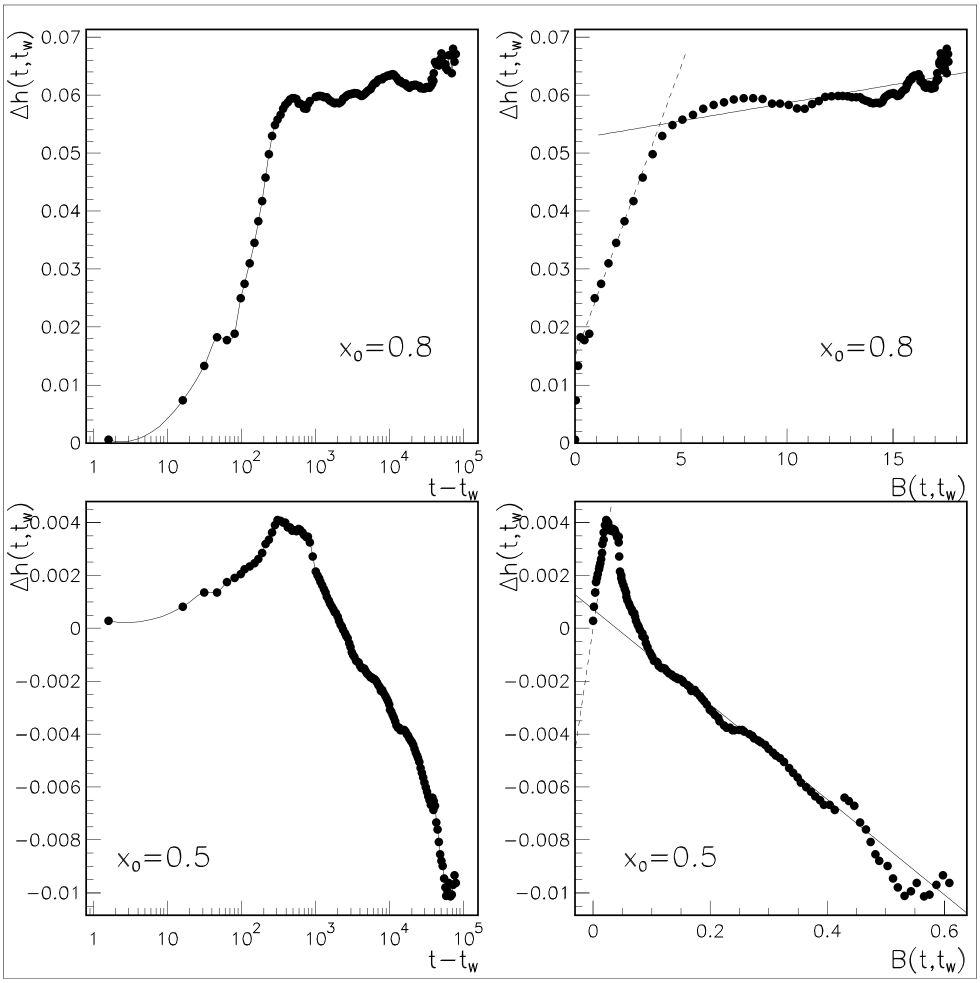

Figure 6. The figures on the left show, as a function of $t - t_w$, the average height difference, $\Delta h(t, t_w) \equiv h_1(t, t_w) - h_0(t)$, of a reference system shaken at a given x_0 and a replica perturbed after t_w by shaking at $x_0 + \Delta x_0$ ($\Delta x_0 = 0.002$). On the right, a check of the generalized fluctuation-dissipation relation (5) is given. The integrated response, $\Delta h(t, t_w)$, is plotted as a function of the displacement of the reference system, $B(t, t_w)$. Systems are shaken at different 'amplitudes' x_0 ($x_0 = 0.8$ (top) and $x_0 = 0.5$ (bottom)), with replicas perturbed after $t_w = 370$. At low x_0, negative responses appear, but, in agreement with equation (5), Δh is asymptotically still approximately linear in B.

Acknowledgments

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